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# Exact Traveling Wave Solutions for Power law and Kerr law non Linearity Using the $\operatorname{Exp}(-\varphi(\xi))$-expansion Method 

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# Exact Traveling Wave Solutions for Power Law and Kerr Law Non Linearity Using the Exp(-ч ( $\xi)$ )-expansion Method 

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#### Abstract

The $\exp (-\varphi(\xi))$-expansion method is used as the first time to investigate the wave solution of the nonlinear Burger equation with power law nonlinearity, the perturbed non-linear Schrodinger equation with kerr law nonlinearity. The proposed method also can be used for many other nonlinear evolution equations. Keywords: $\exp (-\varphi(\xi))$-expansion method, homogeneous balance, travelling wave solutions, solitary wave solutions, the nonlinear burger equation with power law nonlinearity, the perturbed nonlinear schrodinger equation with kerr law nonlinearity.


## I. Introduction

The nonlinear equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, Optics, Plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh - sech method [2]-[4], extended tanh - method [5-7], sine - cosine method [8]-[10], homogeneous balance method [11] and [12], Jacobi elliptic function method [13]-[16], F-expansion method [17]-[19], exp-function method [20]-[21], trigonometric function series method [22], ( $\left.\frac{G^{\prime}}{G}\right)$ - expansion method [23]-[26], the modified simple equation method [27]-[32] and so on.
In the present paper, we shall proposed a new method which is called exp- $\varphi(\xi)$-expansion method to seek traveling wave solutions of nonlinear evolution equations. The main ideas of the proposed method are that the traveling wave solutions of nonlinear evolution equation can be expressed by a polynomial in $\exp -\varphi(\xi)$.
The paper is organized as follows: In section 2, we give the description of exp- $\varphi(\xi)$-expansion method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above and some figures of our results are drawn. In section 4, conclusion are given.

## II. Description of the Exp $(-\varphi(\xi))$-expansion Method

Consider the following nonlinear evolution equation

$$
\begin{equation*}
F\left(u, u_{t}, u_{x}, u_{t t}, u_{x x}, \ldots\right)=0 \tag{2.1}
\end{equation*}
$$

where F is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method

[^0]Step 1. We use the wave transformation

$$
\begin{equation*}
u(x, t)=u(\xi), \quad \xi=x-c t \tag{2.2}
\end{equation*}
$$

where c is a positive constant, to reduce Eq.(1)to the following ODE:

$$
\begin{equation*}
P\left(u, u^{\prime}, u^{\prime \prime}, u^{\prime \prime \prime}, \ldots . .\right)=0 \tag{2.3}
\end{equation*}
$$

where P is a polynomial in $u(\xi)$ and its total derivatives, while ${ }^{\prime}=\frac{d^{\prime}}{d \xi}$.
Step 2. Suppose that the solution of $\operatorname{ODE}(2.3)$ can be expressed by a polynomial in $\exp (-\varphi(\xi))$ as follows

$$
\begin{equation*}
u(\xi)=\alpha_{m}(\exp (-\varphi(\xi)))^{m}+\ldots \ldots, \quad \alpha_{m} \neq 0 \tag{2.4}
\end{equation*}
$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$
\begin{equation*}
\varphi^{\prime}(\xi)=\exp (-\varphi(\xi))+\mu \exp (\varphi(\xi))+\lambda, \tag{2.5}
\end{equation*}
$$

the solutions of ODE (2.5) are
when $\lambda^{2}-4 \mu>0, \mu \neq 0$,

$$
\begin{equation*}
\varphi(\xi)=\ln \left(\frac{-\sqrt{\lambda^{2}-4 \mu} \tanh \left(\frac{\sqrt{\lambda^{2}-4 \mu}}{2}\left(\xi+C_{1}\right)\right)-\lambda}{2 \mu}\right) \tag{2.6}
\end{equation*}
$$

when $\lambda^{2}-4 \mu>0, \mu=0$,

$$
\begin{equation*}
\varphi(\xi)=-\ln \left(\frac{\lambda}{\exp \left(\lambda\left(\xi+C_{1}\right)\right)-1}\right), \tag{2.7}
\end{equation*}
$$

when $\lambda^{2}-4 \mu=0, \mu \neq 0, \lambda \neq 0$,

$$
\begin{equation*}
\varphi(\xi)=\ln \left(-\frac{2\left(\lambda\left(\xi+C_{1}\right)+2\right)}{\lambda^{2}\left(\xi+C_{1}\right)}\right) \tag{2.8}
\end{equation*}
$$

when $\lambda^{2}-4 \mu=0, \mu=0, \lambda=0$,

$$
\begin{equation*}
\varphi(\xi)=\ln \left(\xi+C_{1}\right), \tag{2.9}
\end{equation*}
$$

when $\lambda^{2}-4 \mu<0$,

$$
\begin{equation*}
\varphi(\xi)=\ln \left(\frac{\sqrt{4 \mu-\lambda^{2}} \tan \left(\frac{\sqrt{4 \mu-\lambda^{2}}}{2}\left(\xi+C_{1}\right)\right)-\lambda}{2 \mu}\right) \tag{2.10}
\end{equation*}
$$

where $a_{m}, \ldots, \lambda, \mu$ are constants to be determined later,
Step 3. Substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $\exp (-m \varphi(\xi))$ and equating them to zero, where the positive integer $m$ can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in $\operatorname{ODE}(2.3)$. We obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of $\alpha_{i}$.
Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.3).
a) Example1. The nonlinear Burger equation with power law nonlinearity.

This equation is well known [33] and has the form:

$$
\begin{equation*}
v_{t}+a\left(v^{n}\right)_{x}+b v_{x x}=0, n>1, \tag{2.11}
\end{equation*}
$$

where $a$ and $b$ are nonzero constants. The solutions of Eq.(2.11) have been discussed, the exact solitary wave solutions, the periodic solutions and the rational function solution are obtaines in [33] by means of the extended $\left(\frac{G^{\prime}}{G}\right)$-expansion method. Let us now solve Eq.(2.11) using the $\exp (-\varphi(\xi))$-expansion method. To this end, we use the wave transformation (2.2) to reduce Eq.(2.11) to the ODE and integrating the equation with zero constant of integration:

$$
\begin{equation*}
-c v+a v^{n}+b v^{\prime}=0 . \tag{2.12}
\end{equation*}
$$

Balancing $v^{\prime}$ with $v^{n}$ yields $m=\frac{1}{n-1}, n>1$. Using the transformation

$$
\begin{equation*}
v=u^{\frac{1}{n-1}} \tag{2.13}
\end{equation*}
$$

to reduce Eq.(12) to the following equation

$$
\begin{equation*}
-c(n-1) u+a(n-1) u^{2}+b u^{\prime}=0 \tag{2.14}
\end{equation*}
$$

where $u$ is a new function of $\xi$. Balancing $u^{\prime}$ with $u^{2}$ yields $m=1$. Consequently, Eq.(2.1) has the formal solution

$$
\begin{equation*}
u=\alpha_{0}+\alpha_{1} \exp (-\varphi), \tag{2.15}
\end{equation*}
$$

where $\alpha_{0}$ and $\alpha_{1}$ are constants to be determined, such that $\alpha_{1} \neq 0$. It is easy to see that

$$
\begin{equation*}
u^{\prime}=-\alpha_{1} \exp (-2 \varphi)-\mu \alpha_{1}-\lambda \alpha_{1} \exp (-\varphi), \tag{2.16}
\end{equation*}
$$

substituting Eq.(2.15) and its derivatives in Eq.(2.1) and equating the coefficient of different power's ofexp $(-\varphi(\xi))$ to zero, we get

$$
\begin{gather*}
a(n-1) \alpha_{1}^{2}-b \alpha_{1}=0,  \tag{2.17}\\
-v(n-1) \alpha_{1}+a(n-1)\left(2 \alpha_{0} \alpha_{1}\right)-b \lambda \alpha_{1}=0,  \tag{2.18}\\
-c \alpha_{0}(n-1)+a(n-1) \alpha_{0}^{2}-b \mu \alpha_{1}=0, \tag{2.19}
\end{gather*}
$$

Eqs.(2.1)-(2.19) yield

$$
\begin{equation*}
\alpha_{0}=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}, \alpha_{1}=\frac{b}{a(n-1)} . \tag{2.20}
\end{equation*}
$$

thus the solution is

$$
\begin{equation*}
u=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}+\frac{b}{a(n-1)} \exp (-\varphi) \tag{2.21}
\end{equation*}
$$

Let us now discuse the following case:
Case 1. if $\lambda^{2}-4 \mu>0, \mu \neq 0$. then we deduce from Eq.(2.21) that

$$
\begin{equation*}
\left.u(\xi)=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}+\frac{2 b \mu}{a(n-1)\left[-\sqrt{\lambda^{2}-4 \mu} \tanh \frac{\sqrt{\lambda^{2}-4 \mu}}{2}\right.}\left(\xi+c_{1}\right)-\lambda\right] \quad \tag{2.22}
\end{equation*}
$$

Case 2. if $\lambda^{2}-4 \mu>0, \mu=0$. then we deduce from Eq. that

$$
\begin{equation*}
u(\xi)=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}+\frac{2 b \mu}{a(n-1)\left[\exp \left(\lambda \xi+c_{1}\right)-1\right]} \tag{2.23}
\end{equation*}
$$

Case 3. if $\lambda^{2}-4 \mu=0, \mu \neq 0, \lambda \neq 0$. then we deduce from Eq. that

$$
\begin{equation*}
u(\xi)=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}-\frac{b \lambda^{2}\left(\xi+c_{1}\right)}{2 a(n-1)\left[\lambda\left(\xi+c_{1}\right)+2\right]} . \tag{2.24}
\end{equation*}
$$

Case 4. if $\lambda^{2}-4 \mu=0, \mu=0, \lambda=0$. then we deduce from Eq. that

$$
\begin{equation*}
u(\xi)=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}+\frac{b}{a(n-1)\left[\xi+c_{1}\right]} . \tag{2.25}
\end{equation*}
$$

Case 5. if $\lambda^{2}-4 \mu<0$, then we deduce from Eq. that

$$
\begin{equation*}
u(\xi)=\frac{b \lambda}{2 a(n-1)}+\frac{v}{2 a}+\frac{2 b \mu}{a(n-1)\left[\sqrt{4 \mu-\lambda^{2}} \tan \left(\frac{\sqrt{4 \mu-\lambda^{2}}}{2}\left(\xi+c_{1}\right)\right)-\lambda\right]} \tag{2.26}
\end{equation*}
$$


(a) Eq.(3.22)

(b) Eq.(3.23)

(c) Eq.(3.24)

(d) Eq.(3.25)

(e) Eq.(3.26)

Figure 1 : solution of Eqs.(3.22)-(3.26)

## b) Example2. The perturbed nonlinear Schrodinger equation with Kerr law nonlinearity.

This equation is well-known [34],[35] and has the form:

$$
\begin{equation*}
i u_{t}+u_{x x}+\alpha|u|^{2} u+i\left\{\gamma_{1} u_{x x x}+\gamma_{2}|u|^{2} u_{x}+\gamma_{3}\left(|u|^{2}\right)_{x} u\right\}=0, \tag{2.27}
\end{equation*}
$$

where $\alpha, \gamma_{1}, \gamma_{2}, \gamma_{3}$ are constants such that $\gamma_{1}$ is the third order dispersion, $\gamma_{2}$ is the nonlinear dispersion, while $\gamma_{3}$ is also a version of nonlinear dispersion [36],[37]. Eq.S1 describes the propagation of optical solitons in nonlinear optical fibers that exhibits a Kerr law nonlinearity. Eq.S1 has been discussed in [35] using the first integral method and in [34] using the modified mapping method and its extended. Let us now solve Eq.S1 using the $\exp (-\varphi(\xi))$-expansion method. To this end we seek its traveling wave solution of the form [34],[35]:

$$
\begin{equation*}
u(x, t)=\phi(\xi) \exp [i(k x-\Omega t)], \xi=x-c t \tag{2.28}
\end{equation*}
$$

where $k, \Omega$ and $c$ are constants, while $i=\sqrt{-1}$. Substituting S2 into Eq.S1 and equating the real and imaginary parts to zero, we have

$$
\begin{equation*}
\gamma_{1} \phi^{\prime \prime \prime}+\left(2 k-c-3 \gamma_{1} k^{2}\right) \phi^{\prime}+\left(\gamma_{2}+2 \gamma_{3}\right) \phi^{2} \phi^{\prime}=0 \tag{2.29}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(1-3 \gamma_{1} k\right) \phi^{\prime \prime}+\left(\Omega-k^{2}+\gamma_{1} k^{3}\right) \phi+\left(\alpha-\gamma_{2} k\right) \phi^{3}=0 \tag{2.30}
\end{equation*}
$$

With reference to [34], the two equations (2.29) and (2.30) can be simplified as follows:
Integration Eq.(2.29) and vanishing the constant of integration, we have

$$
\begin{equation*}
\gamma_{1} \phi^{\prime \prime}+\left(2 k-c-3 \gamma_{1} k^{2}\right) \phi+\frac{1}{3}\left(\gamma_{2}+2 \gamma_{3}\right) \phi^{3}=0 \tag{2.31}
\end{equation*}
$$

From Eqs.(2.30) and (2.31) we deduce that

$$
\begin{equation*}
\frac{\gamma_{1}}{1-3 \gamma_{1} k}=\frac{2 k-c-3 \gamma_{1} k^{2}}{\Omega-k^{2}+\gamma_{1} k^{3}}=\frac{\frac{1}{3}\left(\gamma_{2}+2 \gamma_{3}\right)}{\alpha-\gamma_{2} k} . \tag{2.32}
\end{equation*}
$$

From Eq.(2.32), we can obtain $k=\frac{\omega-\alpha \gamma_{1}}{3 \omega \gamma_{1}-\gamma_{1} \gamma_{2}}, \Omega=\frac{\left(1-3 \gamma_{1} k\right)\left(2 k-c-3 \gamma_{1} k^{2}\right)}{\omega}+k^{2}-\gamma_{1} k^{3}$, where $\omega=\frac{1}{3} \gamma_{2}+\frac{2}{3} \gamma_{3}$. Now, Eq.(2.32) is transformed into the following form:

$$
\begin{equation*}
A \phi^{\prime \prime}+B \phi+\omega \phi^{3}=0, \tag{2.33}
\end{equation*}
$$

where $A=\gamma_{1}$ and $B=2 k-c-3 \gamma_{1} k^{2}$. Balancing $\phi^{\prime \prime}$ with $\phi^{3}$ yields $m=1$. Thus, we get the same formulas (2.15). Substituting (2.15) and its derivatives into Eq.(2.33) and equating the coefficients of $\exp (-m \varphi)$ to zero, we get

$$
\begin{gather*}
2 \alpha_{1} A+\omega \alpha_{1}^{3}=0,  \tag{2.34}\\
3 \lambda \alpha_{1} A+3 \alpha_{0} \alpha_{1}^{2} \omega=0  \tag{2.35}\\
A \alpha_{1}\left(\lambda^{2}+2 \mu\right)+3 \alpha_{0}^{2} \alpha_{1} \omega+B \alpha_{1}=0,  \tag{2.36}\\
\lambda \mu \alpha_{1} A+\omega \alpha_{0}^{3}+B \alpha_{0}=0 \tag{2.37}
\end{gather*}
$$

Eqs.(2.34)-(2.37) yields

$$
\begin{equation*}
\alpha_{0}=\mp \lambda \sqrt{\frac{-A}{2 \omega}}, \alpha_{1}= \pm \sqrt{\frac{-2 A}{\omega}} . \tag{2.38}
\end{equation*}
$$

thus the solution is

$$
\begin{equation*}
u=\mp \lambda \sqrt{\frac{-A}{2 \omega}} \pm \sqrt{\frac{-2 A}{\omega}} \exp (-\varphi) \tag{2.39}
\end{equation*}
$$

Let us now discuse the following case:
Case 1. if $\lambda^{2}-4 \mu>0, \mu \neq 0$. then we deduce from Eq.(2.33) that

$$
\begin{equation*}
u(\xi)=\mp \lambda \sqrt{\frac{-A}{2 \omega}} \pm \sqrt{\frac{-2 A}{\omega}}\left[\frac{2 \mu}{-\sqrt{\lambda^{2}-4 \mu} \tanh \frac{\sqrt{\lambda^{2}-4 \mu}}{2}\left(\xi+c_{1}\right)-\lambda}\right], \tag{2.40}
\end{equation*}
$$

Case 2. if $\lambda^{2}-4 \mu>0, \mu=0$. then we deduce from Eq.(2.33) that

$$
\begin{equation*}
u(\xi)=\mp \lambda \sqrt{\frac{-A}{2 \omega}} \pm \sqrt{\frac{-2 A}{\omega}}\left[\frac{\lambda}{\exp \left(\lambda \xi+c_{1}\right)-1}\right] \tag{2.41}
\end{equation*}
$$

Case 3. if $\lambda^{2}-4 \mu=0, \mu \neq 0, \lambda \neq 0$. then we deduce from Eq.(2.33) that

$$
\begin{equation*}
u(\xi)=\mp \lambda \sqrt{\frac{-A}{2 \omega}} \pm \sqrt{\frac{-2 A}{\omega}}\left[\frac{\lambda^{2}\left(\xi+c_{1}\right)}{2\left(\lambda\left(\xi+c_{1}\right)+2\right)}\right] . \tag{2.42}
\end{equation*}
$$

Case 4. if $\lambda^{2}-4 \mu=0, \mu=0, \lambda=0$. then we deduce from Eq.(2.33) that

$$
\begin{equation*}
u(\xi)=\mp \lambda \sqrt{\frac{-A}{2 \omega}} \pm \sqrt{\frac{-2 A}{\omega}}\left[\frac{1}{\xi+c_{1}}\right] \tag{2.43}
\end{equation*}
$$

Case 5. if $\lambda^{2}-4 \mu<0$, then we deduce from Eq.(2.33) that

$$
\begin{equation*}
u(\xi)=\mp \lambda \sqrt{\frac{-A}{2 \omega}}+\frac{v}{2 a} \pm \sqrt{\frac{-2 A}{\omega}}\left[\frac{2 \mu}{\sqrt{4 \mu-\lambda^{2}} \tan \left(\frac{\sqrt{4 \mu-\lambda^{2}}}{2}\left(\xi+c_{1}\right)\right)-\lambda}\right] \tag{2.44}
\end{equation*}
$$


(a) Eq.(3.40)

(b) Eq.(3.41)

(c) Eq.(3.42)

(d) Eq.(3.43)

(e) Eq.(3.44)

Figure 2 : solution of Eqs.(3.40)-(3.44)

## iiI. Conclusions

In this paper, it has been shown that the new $\exp (-(\varphi))$-expansion method is a powerful tool for the nonlinear evolution equations. we can obtained new and more travelling wave solutions for the equations above, such as, the nonlinear Burger equation with power law nonlinearity, the perturbed nonlinear Schrodinger equation with kerr law nonlinearity.. Otherwise, the general solutions of the ODE have been well known for the researchers. Furthermore, the new method can be used for many other nonlinear evolution equations.

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